Overview
If you rub two balloons on your hair or a sample of animal fur, the balloons will become charged. Since they are charged in a like manner, they will repel each other. But what variables might affect the amount of repulsion between the two like-charged balloons? In this activity, you will explore this question using a graphical model. The graph shows the force as a function of the distance of separation between the charged areas of the balloons. There are four lines on the graph, each representing a set of balloons that have been rubbed a different number of times.

The activity will not involve the algebraic manipulation of equations. Rather, you will read coordinate values off the graph and interpret the resulting values. This activity will be challenging because you will have to think about the numerical relationships between the values that you read off the graph. You should have a calculator ready so that you can calculate a few ratios. Most of all, you need to have your thinking cap ready. So turn on your noodle; it’s thinking time!

You will use the Desmos online graphing calculator to complete this activity:
https://www.desmos.com/calculator/uu3jod3qvj

Using the Model
The graph shows the amount of force acting between two balloons as a function of the distance between the two balloons. The force (milliNewton) is plotted along the vertical axis and the distance (cm) is along the horizontal axis. There are four lines on the graph. Each line represents a set of balloons that have been rubbed a different number of times. The colors represent a different number of rubs:

- Blue: 2 rubs
- Red: 5 rubs
- Green: 4 rubs
- Black: 10 rubs

Resizing the Graph
The scale of the graph will change if you scroll when your cursor is inside the graph window. The scale can be reset back to its original settings by clicking on the wrench icon in the top right of the browser window. The original settings are

\[-5 < x < 100\]  \[-1 < y < 10\]

Use the graphical model to answer the following questions. Read carefully. Think constantly.
The graph shows the dependence of the force of repulsion (vertical axis) upon the separation distance (horizontal axis) between the two balloons.

1. From the graph, which statement accurately describes this dependence?
   a. There is a direct relation; as distance increases, the force decreases.
   b. There is an inverse relation; as distance increases, the force decreases.
   c. There is an inverse relation; as distance increases, the force increases.
   d. There is a direct relation; as distance increases, the force increases.

2. Consider the situation in which the balloons were given six vigorous rubs on animal fur. Using the graph, determine the value for the force (in milliNewton) when the balloons are held a distance of ...
   ... 20 cm apart: $F_{\text{when } d = 20 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton
   ... 40 cm apart: $F_{\text{when } d = 40 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton
   ... 60 cm apart: $F_{\text{when } d = 60 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton
   ... 100 cm apart: $F_{\text{when } d = 100 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton

3. Inspect your answers in Question 2 (above). What affect does a doubling of separation distance from 20 cm to 40 cm have upon the force of repulsion between the two balloons?
   a. A doubling of the distance will make the force two times the original value.
   b. A doubling of the distance will make the force four times the original value.
   c. A doubling of the distance will make the force one-half the original value.
   d. A doubling of the distance will make the force one-fourth the original value.

4. Inspect your answers in Question 2 (above). What affect does a tripling of separation distance from 20 cm to 60 cm have upon the force of repulsion between the two balloons?
   a. A tripling of the distance will make the force three times the original value.
   b. A tripling of the distance will make the force nine times the original value.
   c. A tripling of the distance will make the force one-third the original value.
   d. A tripling of the distance will make the force one-ninth the original value.

5. Use your answers to Part b and Part c (above) to predict the value of the repulsive force when the same balloons (rubbed six times) have a separation distance is 30 cm.
   \[ F_{\text{six rubs, } d = 30 \text{ cm}} = \underline{\hspace{2cm}} \text{ milliNewton} \]
   (HINT: 30 cm is one-half the distance of 60 cm.)

Now suppose that two balloons are given 10 vigorous rubs against animal fur and then held varying distances apart.

6. Use the graph to determine the force of repulsion at a distance of ...
   ... 40 cm apart: $F_{\text{10 rubs, } d = 40 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton
   ... 80 cm apart: $F_{\text{10 rubs, } d = 80 \text{ cm}} = \underline{\hspace{2cm}}$ milliNewton
7. Note that the 40 cm and 80 cm of Question 7 are distances with a 1:2 ratio. Use your answers in Question 7 to determine the ratio of the forces for two separation distances that have a 1:2 ratio.

\[
\text{Ratio of } F_{d=40\,\text{cm}} / F_{d=80\,\text{cm}} = \text{______________} \quad \text{(Note: this should be a whole number.)}
\]

8. Now use the ratio from the previous problem to predict the amount of force when the two balloons are separated by a distance of ...

... 20 cm apart: \( F_{10\,\text{rubs, } d=20\,\text{cm}} = \text{______________} \text{ milliNewton} \)

(HINT: 20 cm is one-half the distance of 40 cm.)

... 160 cm apart: \( F_{10\,\text{rubs, } d=160\,\text{cm}} = \text{______________} \text{ milliNewton} \)

(HINT: 160 cm is two times the distance of 80 cm.)

Now we will investigate the effect that a varying number of rubs has upon the amount of repulsive force between the two balloons.

9. Observe the graph below with 10 labeled points placed on different lines.

![Graph with labeled points](image)

Which set of points should be analyzed in order to determine the effect of the number of rubs upon the strength of the force? Why?

a. Points B, C, D, and E; they are located on different lines and have same force.
b. Points A, F, I and J; they are located on different lines and have same distance.
c. Points B, C, D, and E; they are located on different lines and have the same distance.
d. Points E, F, G, and H; they are located on different lines and are on the same diagonal line midway between the axes.
e. Points C, G and I; they are located on the same line and have different forces and different distances.
10. Which statement describes the effect that an increase in the number of rubs has upon the force?
   a. The number of rubs has no effect upon the force between them.
   b. The more two balloons are rubbed, the weaker the force between the balloons.
   c. The more two balloons are rubbed, the greater the force between the balloons.

11. Use the graph to determine the force value at a distance of 40 cm for the following number of rubs:
    Two rubs: $F_{2\text{ rubs}, d=40\text{ cm}} = \underline{\text{milliNewton}}$
    Four rubs: $F_{4\text{ rubs}, d=40\text{ cm}} = \underline{\text{milliNewton}}$
    Six rubs: $F_{6\text{ rubs}, d=40\text{ cm}} = \underline{\text{milliNewton}}$
    Ten rubs: $F_{10\text{ rubs}, d=40\text{ cm}} = \underline{\text{milliNewton}}$

12. Compare the force between balloons when they have been rubbed four times to the force between balloons when they have been rubbed two times. Calculate the ratio of these two forces. (HINT: the answer will be a whole number integer.)
    \[ \frac{F_{4\text{ rubs}}}{F_{2\text{ rubs}}} = \underline{\text{ }} \]

13. Compare the force between balloons when they have been rubbed six times to the force between balloons when they have been rubbed two times. Calculate the ratio of these two forces. (HINT: the answer will be a whole number integer.)
    \[ \frac{F_{6\text{ rubs}}}{F_{2\text{ rubs}}} = \underline{\text{ }} \]

14. Compare the force between balloons when they have been rubbed ten times to the force between balloons when they have been rubbed two times. Calculate the ratio of these two forces. (HINT: the answer will be a whole number integer.)
    \[ \frac{F_{10\text{ rubs}}}{F_{2\text{ rubs}}} = \underline{\text{ }} \]

15. Use the answers to Question 12 through Question 14 to complete the following statements describing the effect of varying number of rubs upon the force:
    • If the number of times the balloons are rubbed is doubled, the force between the balloons will \underline{increase} (increase, decrease) by a factor of \underline{ }.
    • If the number of times the balloons are rubbed is tripled, the force between the balloons will \underline{increase} (increase, decrease) by a factor of \underline{ }.
    • If the number of times the balloons are rubbed is increased by a factor of 5, the force between the balloons will \underline{increase} (increase, decrease) by a factor of \underline{ }.

16. Which statement accurately describes the effect of the number of times the balloons are rubbed upon the amount of force between the two balloons?
   a. The force is directly proportional to the number of rubs.
   b. The force is inversely proportional to the number of rubs.
   c. The force is directly proportional to the square of the number of rubs.
   d. The force is inversely proportional to the square of the number of rubs.
17. Use the rule discovered in **Question 15** to predict the amount of force between the balloons when they have been rubbed twenty times and are held a distance of 40 cm apart.

\[ F_{20 \text{ rubs}}, d = 40 \text{ cm} = \underline{\text{milliNewton}} \]

18. Use the rule discovered in **Question 15** and information from the graph to predict the amount of force between the balloons when they have been rubbed twenty times and are held a distance of 60 cm apart.

\[ F_{20 \text{ rubs}}, d = 60 \text{ cm} = \underline{\text{milliNewton}} \]